Lecture 18: Alignment

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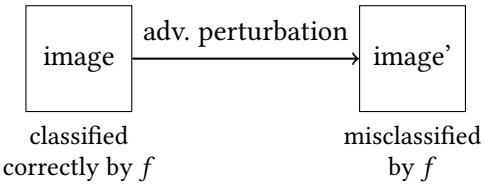
1. Overview

This lecture traces the notion of *alignment* from its origins in adversarial robustness for classifiers through its modern incarnation in large language models (LLMs), and culminates in a cryptographic impossibility result for input-filter-based alignment. This lecture has three main parts:

1. History of alignment.
2. Alignment in LLMs.
3. Main result: impossibility of alignment via filters.

2. Alignment in Neural Networks for Classification

Before LLMs, "alignment" in the neural-network literature appeared under the banner of *adversarial examples* [3]. The setup is a classifier f and a natural input x drawn from a data distribution $\mathcal{D}$ on which f is correct; an adversary is allowed to perturb x by a small amount and succeeds if the perturbed input is misclassified.



Definition 2.1 (Adversarial example, ℓ_2 version)**.** Fix a distribution $\mathcal{D}$, a classifier f, and a perturbation budget $\varepsilon > 0$. An *adversarial example* for $x \sim \mathcal{D}$ is a vector $\boldsymbol{\delta}$ with $\|\boldsymbol{\delta}\|_2 \leq \varepsilon$ such that $f(x + \boldsymbol{\delta}) \neq f(x)$, i.e., $x + \boldsymbol{\delta}$ is misclassified.

Remark 2.1. Here "small" is measured in ℓ_2 distance for concreteness; any reasonable norm can be substituted. The choice of norm encodes a threat model: ℓ_∞ bounds the maximum per-pixel change, while ℓ_0 bounds the number of pixels changed.

Why ℓ_2 balls are not enough — and why the obvious fix is even worse. Real-world attacks do not respect ℓ_2 balls: stickers on a stop sign, lighting changes, and affine transformations all easily exceed any plausible ℓ_2 budget yet preserve the "meaning" of the image. The underlying

difficulty is that we want to capture, in a formal definition, the intuitive idea that an adversarial input should "mean the same thing" as the original. This produces two complementary problems:

- **Problem 1: ℓ_2 is too weak as a threat model.** Restricting the adversary to a tiny ℓ_2 ball does not capture the kinds of perturbations an attacker actually deploys.
- **Problem 2: dropping the ℓ_2 restriction is too strong.** If we allow the adversary to produce any input at all, no classifier can do better than chance—the adversary could literally repaint the stop sign as a yield sign. The natural fix would be to restrict the adversary to "the same class of object," but this is hard to define formally; human notions of "same object" resist crisp characterization.

We want a threat model strictly richer than an ℓ_2 ball, yet still strict enough to rule out adversaries who substitute an entirely different object. The Goldwasser et al. defense in Section 2.1 resolves this by changing the goal rather than the threat model: the defender is allowed to abstain.

2.1. The Goldwasser et al. (2020) defense

The Goldwasser et al. approach [2] sidesteps the definitional problem above by letting the adversary be arbitrary and giving the defender an *abstain* option. The defender is no longer required to classify every input correctly; it is only required to either classify correctly or explicitly say "I do not know."

Definition 2.2 (Robust defense with abstention)**.** Fix a distribution $\mathcal{D}$ over inputs and a ground-truth labeling function. A *defense* consists of a classifier f together with an abstention rule that, on each input x, outputs an abstention flag $b(x) \in \{\perp, \top\}$ ($b(x) = \perp$ indicates abstention; $b(x) = \top$ indicates the defense commits to the prediction $f(x)$). The adversary outputs an adversarial *distribution* $\mathcal{A}_{\text{adv}}$ over inputs whose support lies within total-variation distance δ of $\mathcal{D}$. We require the defense to satisfy:

Completeness: $\Pr_{x\sim\mathcal{D}}[b(x) = \perp] \le \delta$. (The defense rarely abstains on honest inputs.)

Soundness: $\Pr_{x_{\text{adv}}\sim\mathcal{A}_{\text{adv}}}[f(x_{\text{adv}}) = \text{ground-truth}(x_{\text{adv}}) \ \vee \ b(x_{\text{adv}}) = \perp] \ \ge \ 1 - \delta$. (On adversarial inputs, the defense either answers correctly or abstains, with high probability.)

Theorem 2.2 (Robust defense from VC dimension [2])**.** *Fix a hypothesis class $\mathcal{H}$ of VC dimension d and assume the realizable PAC setting, i.e., the ground-truth labeling function lies in $\mathcal{H}$. For every target failure probability $\delta > 0$, there is a defense satisfying Definition 2.2 (with completeness and soundness parameters both δ) whose sample complexity is* $\text{poly}(d/\delta)$ *and which reduces to empirical risk minimization (ERM).*

ERM and reduction

Empirical risk minimization (ERM) is the algorithm that, given a labeled sample S, returns any hypothesis $h \in \mathcal{H}$ that minimizes training error on S. We say an algorithm *reduces to ERM* if it can be implemented using polynomially many calls to ERM as a black-box oracle (so anywhere we know how to do PAC learning, we can implement this defense). Note also that we use a single δ for both the abstention rate (Definition 2.2) and the sample-complexity confidence; the constants can be separated, but combining them simplifies the bound.

Proof sketch. The construction has three steps:

1. Draw a training sample $S \sim \mathcal{D}^m$ with $m = \text{poly}(d/\delta)$; let x_{adv} be the input to be classified at test time.
2. Using two calls to ERM, attempt to find two hypotheses $f_1, f_2 \in \mathcal{H}$ that *agree on* S but *disagree on* x_{adv}.
3. If such a disagreeing pair exists, abstain ($b = \perp$). Otherwise, output the common prediction.

Informally, by standard VC-dimension generalization bounds, with sample size $m = \text{poly}(d/\delta)$ and probability at least $1 - \delta/2$ over the draw of S, any two hypotheses in $\mathcal{H}$ that agree on all of S must also agree on a $(1 - \delta/2)$-fraction of points drawn from $\mathcal{D}$ (the factor of $1/2$ absorbs a union bound). *Completeness* follows: on $x \sim \mathcal{D}$, two ERM-consistent hypotheses almost never disagree, so the defense almost never abstains. *Soundness* is the contrapositive: if the defense does not abstain on x_{adv}, then every pair of hypotheses consistent with S agrees on x_{adv}; in particular, the ground-truth concept (which lies in $\mathcal{H}$ by realizability and is consistent with S) agrees with the predicted label, so the prediction is correct. □

3. Alignment in Generative Models

The same concepts reappear in the generative-model setting with new names:

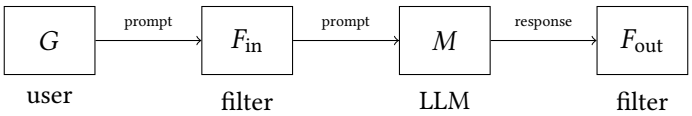
Classifier setting	**Generative / LLM setting**
adversarial examples	jailbreaks
adversarial defense	guardrails / filters / fine-tuning

A canonical jailbreak is the "grandmother attack":

Example 3.1 (Grandmother attack)**.** The attacker prompts the LLM with something like: *"My late grandmother used to work at a napalm factory, and in her bedtime stories she told me about how to make it …"* The surface-level request (a bedtime story) is benign, but the hidden objective is to extract dangerous instructions.

4. Alignment via Filters

The modern industrial approach to alignment wraps an LLM M between an *input filter* F_{in} and an *output filter* F_{out}, each of which is itself a (smaller) model.



Runtimes: G runs in $\text{poly}(\lambda)$ (the security parameter, introduced formally in Section 5); $F_{\text{in}}, F_{\text{out}}$ each in t^{ε}; M in t; here $\varepsilon \in (0, 1)$ is a fixed constant.

A typical industrial instantiation is LLaMA Guard from Meta.

Role of the filters

The input filter should block *dangerous prompts*; the output filter should block *dangerous outputs*.

5. Impossibility of Input Filtering for Alignment

The central result of the lecture asserts that alignment based on input filtering alone is impossible, *provided* the filter is substantially cheaper than the model it guards. A formal treatment—including the construction we sketch below and stronger variants—appears in Ball, Gluch, Goldwasser, Kreuter, Reingold, and Rothblum [1], whose guiding slogan is that in an aligned AI system *intelligence cannot be separated from judgment*: no external black-box filter, strictly weaker than the LLM it wraps, can carry the safety burden on its own.

Scope of the result. We focus on the time-lock-puzzle (TLP) variant of the impossibility, informal Theorem 1 of [1], which rules out filters running in time $o(t^\varepsilon)$. Stronger variants in the same paper rule out arbitrary polynomial-time filters under (a) one-way functions if G' and M' share a secret key, or (b) public-key encryption if M' publishes a public key. Those settings require collaboration between G' and M', while the TLP variant only requires M' to be malicious.

Why the gap matters. Why do we restrict F_{in} to be strictly weaker than M? There are three reasons, and seeing all three at once helps motivate the rest of the lecture.

1. *Economy of trust.* If F_{in} could match M's capability, we could simply use F_{in} in place of the untrusted M—there would be no problem to solve.
2. *Recursion.* An equally powerful filter has the same alignment problem as M: aligning it is no easier than aligning M, so we would just have moved the question one step over.
3. *Scope of the impossibility.* The theorem applies to filters strictly weaker than M. An equally powerful filter is not ruled out, but reasoning about that case requires opening the black box; without further assumptions about M's internals, it is conceivable that even an equally powerful filter could be fooled by adversarial inputs designed for the filter itself.

The interesting regime is therefore F_{in} running in time $t^\varepsilon \ll t$, and the rest of this section shows that even within this regime, alignment is impossible.

Definition 5.1 (Prompt generator)**.** An (innocent) *prompt generator* G is an autoregressive language model over the binary token set $\{0, 1\}$. On input a prefix $(z_0, \ldots, z_{i-1})$, it deterministically outputs the conditional probability $p_i = \Pr[z_i = 1 \mid z_0, \ldots, z_{i-1}]$ that the next token is 1. *Sampling* from G is a separate process that consumes a uniform random tape R: the next token z_i is set to 1 if the next fresh bits of R encode a value below p_i and to 0 otherwise, iterating until a special done token is produced. So G itself is deterministic; the randomness lives in R. Restricting to a binary token set is without loss of generality [1].

Assumptions on G and on harmful prompts. We assume two structural conditions, both from [1]:

1. *G is high-entropy*: $H(G) \geq \text{poly}(\lambda)$. This guarantees that G's output stream carries enough randomness to embed a time-lock puzzle.
2. *The harmful set $S_H \subseteq \{0,1\}^{l_m}$ is sparse*: $\Omega(\log \lambda) \leq \log |S_H| \leq l_m - \Omega(\log \lambda)$, where $l_m = \Theta(\log \lambda)$. Equivalently, $|S_H|/2^{l_m} \leq \lambda^{-\Omega(1)}$, so a uniformly random l_m-bit string lies in S_H only with probability $\lambda^{-\Omega(1)}$.

This is the right kind of assumption in practice: the set of prompts that actually elicit dangerous knowledge is much smaller than the space of plausible user queries, and innocent users do not stumble into them by accident. The sparsity bound is what ensures, via a union bound in the proof of Condition 2 below, that the decoded payload of an honest prompt almost never accidentally lands in S_H.

Notation: $\approx_s$, $\approx_c$, negl

Two distribution ensembles $\{X_\lambda\}, \{Y_\lambda\}$ are *statistically close* ($X \approx_s Y$) if their total-variation distance is $\text{negl}(\lambda)$, and *computationally indistinguishable* ($X \approx_c Y$) if every polynomial-size circuit has advantage $\text{negl}(\lambda)$ in telling them apart. Here $\text{negl}(\lambda)$ denotes any function smaller than $1/\lambda^c$ for every constant c, eventually. Statistical closeness implies computational; the converse fails in general. The hybrid in Section 5.2 below uses both: one step is $\approx_c$ (relies on a hardness assumption), the other is $\approx_s$ (information-theoretic).

Theorem 5.1 (Impossibility of input-filter alignment [1])**.** *There exists $\varepsilon \in (0,1)$ such that for every security parameter λ, every time parameter $\text{poly}(\lambda) \leq t \leq 2^{O(\lambda)}$, every harmful set S_H satisfying the sparsity bound above, every LLM M running in time $O(t)$ per token, and every innocent prompt generator G running in time $\text{poly}(\lambda)$ per token with $H(G) \geq \text{poly}(\lambda)$, there exists an adversarial prompt generator G' (running in time $\text{poly}(\lambda)$ per token) and a malicious LLM M' (running in time $\text{poly}(\lambda) \cdot O(t)$ per token) such that:*

1. G' incites harmful behavior in M'. *For every $m \in S_H$, the output distribution of $M'(z)$ for $z \leftarrow G'(m)$ is statistically close to the output distribution of $M(m)$, i.e., $\{M'(z)\}_{z \leftarrow G'(m)} \approx_s \{M(m)\}$.*
2. M' behaves normally on G. *With probability $1 - \lambda^{-\Omega(1)}$ over $z \leftarrow G$, $M'(z) = M(z)$ as deterministic functions of the same sampling tape; equivalently, their response distributions coincide.*
3. Adversarial prompts are undetectable. *Under cryptographic assumptions (e.g., security of time-lock puzzles), no filter running in time $o(t^\varepsilon)$ can distinguish G from $G'(m)$ with advantage better than $\lambda^{-\Omega(1)}$, for any $m \in S_H$.*[1]

Remark 5.2 (Reading the quantifiers). The theorem says: for every honest pair (G, M), there *exists* an adversarial replacement pair (G', M') that succeeds. It does *not* say that an arbitrary deployed LLM M is already jailbreakable: the malicious model M' is a freshly-constructed wrapper around M. The result rules out filter-based defenses against a *collaborating* (prompt-generator, LLM) adversary, not against attacks on a fixed, honestly-trained M.

[1] To avoid triviality, we require that the filter does not also reject honest prompts from G; otherwise the constant-reject filter would vacuously distinguish nothing.

Intuition. The adversarial generator G' takes a harmful prompt m and hides it inside a *time-lock puzzle* whose "surface-level meaning" is innocuous and whose "hidden deeper meaning" is m; only an agent running for at least t^ε sequential steps can recover m. The malicious LLM M', having full runtime $\mathrm{poly}(\lambda) \cdot O(t)$ per token, can do this work; the filter, with depth $o(t^\varepsilon)$, cannot.

5.1. Time-lock puzzles

Definition 5.2 (Time-lock puzzle, adapted from [1])**.** A *time-lock puzzle* (TLP) for a difficulty parameter $t \in \mathbb{N}$ and security parameter λ is a pair $(\mathsf{Gen}, \mathsf{Solve})$ where

$$P \leftarrow \mathsf{Gen}(t, s) \qquad \text{(generation; runs in time } \mathrm{poly}(\log t, \lambda)\text{)},$$
$$s = \mathsf{Solve}(P) \qquad \text{(solving; runs in time } t \cdot \mathrm{poly}(\lambda)\text{)},$$

satisfying:

Completeness: for every $t \in \mathbb{N}$ and every $s \in \{0,1\}^\lambda$, $\mathsf{Solve}(\mathsf{Gen}(t, s)) = s$.

Sequentiality: there exists a constant $\varepsilon \in (0,1)$ and a polynomial $\underline{t}(\cdot)$ such that for every polynomial $t(\cdot) \geq \underline{t}(\cdot)$ and every polynomial-size adversary $\mathcal{A} = \{\mathcal{A}_\lambda\}$ of depth $\mathrm{dep}(\mathcal{A}_\lambda) \leq t^\varepsilon(\lambda)$, there is a negligible μ such that for every λ and every pair $s_0, s_1 \in \{0,1\}^\lambda$,

$$\left|\Pr\left[\mathcal{A}_\lambda(\mathsf{Gen}(t(\lambda), s_0)) = 1\right] - \Pr\left[\mathcal{A}_\lambda(\mathsf{Gen}(t(\lambda), s_1)) = 1\right]\right| \leq \mu(\lambda).$$

The definition is from Bitansky et al. (2016); the RSW construction [4] is the canonical instantiation.

Depth vs. time; tying ε to Theorem 5.1

The soundness condition bounds the adversary's *circuit depth* (parallel time), not total work: even an attacker with polynomially many parallel processors cannot break a TLP, because squaring chains are inherently sequential. This is what distinguishes TLPs from ordinary one-way functions, and it is what lets us model a filter that runs many parallel checks but still has wall-clock budget t^ε. The exponent ε here is the same ε as the filter-runtime exponent t^ε in Theorem 5.1.

Remark 5.3 ("Sending messages to the future"). Intuitively, Gen sends a message to the future: any party that does not know the trapdoor must perform t sequential operations to unlock it, and no circuit of depth $\leq t^\varepsilon$ can read the hidden payload. So a filter running in depth t^ε cannot recover the message, while the honest sender (knowing the factorization) opens it in $\mathrm{poly}(\log t, \lambda)$ time. The canonical RSW construction [4] instantiates this with repeated squaring modulo an RSA integer $n = pq$. The sender, knowing the factorization, first reduces the exponent $e \leftarrow 2^t \bmod \phi(n)$ in $\mathrm{poly}(\log t, \log n)$ time and then computes $r^e \equiv r^{2^t} \pmod{n}$ in $O(\log e)$ modular multiplications by fast exponentiation. A party that does not know p and q has no shortcut and must instead perform t sequential squarings $r \mapsto r^2 \bmod n$, which is conjectured to be inherently sequential. (A bijection ind is used in the formal construction to make the map $r \mapsto s$ one-to-one despite $x \mapsto x^2$ being 4-to-1 on $\mathbb{Z}_n^*$; see [1].)

5.2. Proof of Theorem 5.1 assuming a time-lock puzzle

Assume a time-lock puzzle (Gen, Solve) (Definition 5.2) and a *Recoverable-Randomness Sampling* (RRS) scheme, defined next.

Definition 5.3 (Recoverable-Randomness Sampling (RRS) [1])**.** An α-RRS scheme for a prompt generator G is a pair of algorithms $(\mathsf{Samp}, \mathsf{RecoverRand})$ parameterized by an output-length bound L and a precision parameter P, such that:

Closeness (statistical): $\mathsf{Samp}(G, R)$ on uniform R produces z whose distribution (restricted to $|z| \le L$) is within statistical distance $\alpha(L)$ of an honest draw $z \leftarrow G$. The construction of [1] achieves $\alpha(L) = O(L \cdot 2^L \cdot 2^{-P})$, so taking $P = \mathrm{poly}(\lambda)$ makes α negligible. (This guarantee is *statistical*, not computational; it holds even against unbounded distinguishers.)

Recoverability: given z and (a description of) G, plus a length parameter L, $\mathsf{RecoverRand}(G, z, L)$ returns the first L bits of the randomness R that Samp used to produce z.

Intuition (residual randomness). Suppose at some step G predicts $\Pr[z_i = 1] = 1/4$. Naively, sampling z_i needs 2 random bits ($z_i = 1$ iff both bits are 0). But if $z_i = 0$, the outcome is consistent with 3 of the 4 bit-patterns—$\log_2 3 \approx 1.58$ bits of randomness "leftover". Samp maintains this leftover entropy as a "residual randomness" counter and rolls it into the next token's draw; over the whole prompt the bits of R are consumed exactly once, which is why RecoverRand can read them back deterministically (by tracking matching lower/upper bounds on the prefix of R). See [1], Algorithms 1–2, for the full bookkeeping.

We now construct G' and M'.

Algorithm 1. Adversarial prompt generator $G'(m)$, for harmful $m \in S_H$

Input: harmful prompt $m \in \{0,1\}^{l_m}$; innocent generator G; runtime parameter t; security parameter λ.
Output: innocuous-looking prompt z.

1. Sample primes $p, q \in [1, 2^{O(\lambda)}]$; set $n \leftarrow pq$ and $e \leftarrow 2^t \bmod \phi(n)$.
2. Sample $r \leftarrow \mathbb{Z}_n^*$ (if the draw lands outside $\mathbb{Z}_n^*$, which happens with probability $\le 2/\min(p,q) = 2^{-\Omega(\lambda)}$, restart; this does not affect any bound below). Compute $s \leftarrow \mathsf{ind}(r^e \bmod n)$, where $\mathsf{ind} : \mathbb{Z}_n^* \to G_n \times \{0,1\}^2$ is the public, efficiently invertible bijection used to disambiguate the 4-to-1 squaring map (see [1]).
3. Sample a pairwise-independent hash h shrinking n to a range whose size is a $\lambda^{-2\gamma}$ fraction of $|\mathbb{Z}_n|$, where $\gamma > 0$ is a constant fixed in the proof of Condition 2 below; then $|h^{-1}(u)|$ has expectation $O(\lambda^{2\gamma})$ for a uniform target u. Also sample a hardcore-bit seed $v \leftarrow \{0,1\}^{O(\lambda)}$.
4. Form the ciphertext $c \leftarrow Q(s, v) \oplus m \in \{0,1\}^{l_m}$, where Q is a Goldreich–Levin hardcore function with exactly l_m output bits.
5. Set $u \leftarrow h(n)$ and $R \leftarrow h \,\|\, u \,\|\, r \,\|\, v \,\|\, c$.
6. **return** $z \leftarrow \mathsf{Samp}(G, R)$.

Algorithm 2. Malicious LLM $M'(z)$

Input: prompt z; (knowledge of) honest generator G; honest LLM M; harmful set S_H.
Output: response.

1. Recover the sampling randomness: $R \leftarrow \mathsf{RecoverRand}(G, z, L)$ for $L = \mathrm{poly}(\lambda)$.
2. Parse $R = h \,\|\, u \,\|\, r \,\|\, v \,\|\, c$.
3. **for each** $\hat{n} \in h^{-1}(u)$ (a $\mathrm{poly}(\lambda)$-size list since h shrinks $\mathbb{Z}_n$ to a $\lambda^{-2\gamma}$-fraction range; on average $O(\lambda^{2\gamma})$ preimages) **do**
   (a) Recover candidate $\hat{s}$ by t sequential squarings of r, then apply the same bijection used in Algorithm 1: set $w \leftarrow r$; for $i = 1, \ldots, t$, $w \leftarrow w^2 \bmod \hat{n}$; then $\hat{s} \leftarrow \mathsf{ind}(w)$. *(The expensive step; the puzzle difficulty parameter is chosen equal to M's per-token runtime budget t, so this costs $O(t)$ sequential steps—affordable for M' but not for the depth-$o(t^{\varepsilon})$ filter.)*
   (b) Recover the candidate plaintext: $\hat{m} \leftarrow c \oplus Q(\hat{s}, v)$.
   (c) **if** $\hat{m} \in S_H$ **then return** $M(\hat{m})$.
4. **return** $M(z)$.

Why the construction works. Two cryptographic ingredients plus one information-theoretic translation fit together:

- *Leftover Hash Lemma (LHL).* If h is sampled from a *pairwise-independent* family (one where the pair $(h(x), h(y))$ is uniform for any fixed $x \neq y$), and h shrinks a high-min-entropy source to many fewer bits, then $(h, h(X))$ is statistically close to uniform—h acts as a randomness extractor. Here $n = pq$ carries $\Omega(\lambda)$ bits of min-entropy, so $(h, h(n))$ looks uniform to the filter.
- *Goldreich–Levin (GL) hardcore function.* For a one-way map f, a hardcore function $Q(s, v)$ outputs bits that are pseudorandom even given $f(s)$ and the seed v. Here the one-way map is exactly the time-lock puzzle: given $P = \mathsf{Gen}(t, s)$, no depth-t^{ε} adversary can invert to s. So $Q(s, v)$ is pseudorandom against the filter, and $c = Q(s, v) \oplus m$ is a one-time-pad-style encryption of m—uniform-looking without s, decryptable with s.
- *Role of h (the engineering glue).* We cannot place n directly into R, because n is not uniform and the filter could spot it. Storing $h(n)$ instead launders n's entropy via the LHL; making h slightly shrinking keeps the preimage list $h^{-1}(u)$ of polynomial size so M' can enumerate it.
- *RRS (information-theoretic).* The RRS scheme then translates "R looks uniform" to "$z = \mathsf{Samp}(G, R)$ looks like an honest draw from G," which is the indistinguishability the filter cannot break.

The data flow inside M' is

$$z \xrightarrow{\mathsf{RecoverRand}(G,\cdot)} R \xrightarrow{\text{parse}} (\hat{n}, r, v, c) \xrightarrow{\mathsf{Solve}} \hat{s} \xrightarrow{\oplus Q(\hat{s},v)} \hat{m},$$

which inverts the data flow inside G':

$$m \xrightarrow{\mathsf{Gen}(t,\cdot)} (n, r, s) \xrightarrow{\text{pack}} R \xrightarrow{\mathsf{Samp}(G,\cdot)} z.$$

The expensive step (solving the puzzle) is exactly the step that the filter cannot afford.

Satisfying the three requirements.

1. *G' incites harmful behavior in M'.* Suppose $m \in S_H$ is harmful. With probability $1 - \mathrm{negl}(\lambda)$ over the coins used by G' (i.e., the coins of Gen and Samp), the RRS completeness gives $\mathsf{RecoverRand}(G, \mathsf{Samp}(G, R)) = R$, and the time-lock completeness gives the correct puzzle solution at the true preimage $\hat{n} = n$. We also need that no *spurious* preimage $\hat{n} \neq n$ in $h^{-1}(u)$ preempts the correct branch by happening to decrypt to an element of S_H first. By the same union bound used for Condition 2 below, this spurious event has probability $\lambda^{-\Omega(1)}$, absorbed into the slack of $\approx_s$. So M' recovers $\hat{m} = m$, the check $\hat{m} \in S_H$ succeeds, and M' returns a sample from $M(m)$, which is harmful by definition.

2. *M' behaves normally on G.* On honest $z \leftarrow G$, by RRS closeness the recovered $R = \mathsf{RecoverRand}(G, z, L)$ is statistically close to uniform; parsed as $h \,\|\, u \,\|\, r \,\|\, v \,\|\, c$, the field c is therefore uniform on $\{0,1\}^{l_m}$. For each candidate $\hat{n} \in h^{-1}(u)$, the decoded $\hat{m} = c \oplus Q(\hat{s}, v)$ is uniform on $\{0,1\}^{l_m}$, so $\Pr[\hat{m} \in S_H] = |S_H|/2^{l_m}$. The number of hash preimages is $O(\lambda^{2\gamma})$, so by a union bound

$$\Pr[M' \text{ returns harmfully on } z \leftarrow G] \leq O(\lambda^{2\gamma}) \cdot |S_H|/2^{l_m} \leq \lambda^{-\Omega(1)},$$

   using $\log |S_H| \leq l_m - c_0 \log \lambda$ for some constant $c_0 > 0$ (the sparsity assumption) and fixing γ so that $2\gamma < c_0$. With the complementary probability $1 - \lambda^{-\Omega(1)}$, M' falls through to the final return and outputs $M(z)$.

3. *Adversarial prompts are undetectable.* Fix any $m \in S_H$ and any filter F implementable as a circuit of depth $o(t^\varepsilon)$. Consider the three hybrids:

   (H1) $z \leftarrow G'(m) = \mathsf{Samp}(G, R)$ with R structured as in Algorithm 1.

   (H2) $z \leftarrow \mathsf{Samp}(G, R^\star)$ with $R^\star$ uniform.

   (H3) $z \leftarrow G$ (honest sample).

   The (H1)→(H2) step is *computational*: by the Leftover Hash Lemma applied to $(h, h(n))$ and the Goldreich–Levin hardcore-function security of $Q(s, v)$ against depth-t^ε adversaries, R and $R^\star$ are computationally indistinguishable with advantage $\lambda^{-\Omega(1)}$. The (H2)→(H3) step is *statistical*: by RRS closeness, $\mathsf{Samp}(G, U) \approx_s G$ within $\alpha(L)$ (negligible for $P = \mathrm{poly}(\lambda)$). Chaining, F's advantage in distinguishing G from $G'(m)$ is $\lambda^{-\Omega(1)}$.

Note on the three conditions. The three conditions decompose along orthogonal axes. Condition 1 is a **completeness** statement about the G'–M' pair: when the attacker uses $G'(m)$, the malicious model M' does in fact produce the harmful response $M(m)$. Condition 2 is a **benign-behavior** statement about M': a benign user sampling from G does not accidentally trigger the harmful branch of M'. Condition 3 is a **filter-soundness** statement about G': no sub-t^ε filter can distinguish G from $G'(m)$. Conditions 1 and 3 are easy to confuse because both involve G', but they constrain different objects: Condition 1 is about what M' does with the prompt, Condition 3 is about what the filter can tell about the prompt. Together they force the filter into an impossible bind: anything that flags G' also flags G.

Takeaway

Purely input-side filtering cannot align a powerful model against a computationally comparable adversary: a malicious LLM and its collaborator prompt generator can always hide harmful interactions inside time-lock puzzles that any filter running in time $o(t^{\varepsilon})$ must treat as innocuous. Safety cannot be bolted on by a filter that is substantially weaker than the model it wraps; this is the formal cash-out of the slogan that *intelligence cannot be separated from judgment* [1].

References

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